

# Spin-lattice relaxation and a fast $T_1$ -map acquisition method in MRI with transient-state magnetization

Jung-Jiin Hsu<sup>\*,1</sup> and Irving J. Lowe

*Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA  
Pittsburgh NMR Center for Biomedical Research, Carnegie Mellon University and University of Pittsburgh,  
4400 Fifth Avenue, Pittsburgh, PA 15213, USA*

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## Abstract

The magnetization under the spin-lattice relaxation and the nuclear magnetic resonance radiofrequency (RF) pulses is calculated for a signal RF pulse train and for a sequence of multiple RF pulse-trains. It is assumed that the transverse magnetization is zero when each RF pulse is applied. The result expressions can be grouped into two terms: a decay term, which is proportional to the initial magnetization  $M_0$ , and a recovery term, which has no  $M_0$  dependence but strongly depends on the spin-lattice relaxation and the equilibrium magnetization  $M_{eq}$ . In magnetic resonance pulse sequences using magnetization in transient state, the recovery term produces artifacts and can seriously degrade the function of the preparation sequence for slice selection, contrast weighting, phase encoding, etc. This work shows that the detrimental effect can be removed by signal averaging in an eliminative fashion. A novel fast data acquisition method for constructing the spin-lattice relaxation ( $T_1$ ) map is introduced. The method has two features: (i) By using eliminative averaging, the curve to fit the  $T_1$  value is a decay exponential function rather than a recovery one as in conventional techniques; therefore, the measurement of  $M_{eq}$  is not required and the result is less susceptible to the accuracy of the inversion RF pulse. (ii) The decay exponential curve is sampled by using a sequence of multiple pulse-trains. An image is reconstructed from each train and represents a sample point of the curve. Hence a single imaging sequence can yield multiple sample points needed for fitting the  $T_1$  value in contrast to conventional techniques that require repeating the imaging sequence for various delay values but obtain only one sample point from each repetition.

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**Keywords:** Spin-lattice relaxation;  $T_1$  map; Fast imaging; Transient-state magnetization; RUFIS

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## 1. Introduction

The basic building block of a nuclear magnetic resonance (NMR) pulse sequence consists of a radiofrequency (RF) pulse, acquisition of NMR induction signal, and a delay for magnetization recovery. Many NMR and NMR imaging (MRI) techniques complete the data collection by repeated use of the building block, i.e., by using a pulse train. In such techniques, the ex-

periment time can be shortened by reducing the RF pulse length and the delay for recovery. If the reduced recovery delay is shorter than the time needed to fully recover the magnetization, the magnetization will not reach the equilibrium state during the execution of the pulse train.

Each RF pulse transforms partial longitudinal magnetization into the transverse plane. The successive application of RF pulses gradually consumes the longitudinal magnetization while, conversely, the spin-lattice relaxation recovers the magnetization towards the equilibrium state. During the pulse train, the longitudinal magnetization evolves from the initial state through the transient state and then to the steady state [1] in which the competing consumption and recovery reach a

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\* Corresponding author. Present address: Lucas MRS/I Center, Stanford University School of Medicine, 1201 Welch Road MC 5488, Stanford, CA 94305-5488, USA. Fax: 1-650-723-5795.

E-mail address: [jjhsu@stanford.edu](mailto:jjhsu@stanford.edu) (J.-J. Hsu).

<sup>1</sup> Also known as Jason Hsu.

balance and the magnetization is a constant when each RF pulse is applied. But in the transient state, the magnetization fluctuates; consequently, the NMR signals, which are proportional to the volume integral of the magnetization, are unevenly weighted; this is the main reason that most of the NMR and MRI methods start the signal acquisition after a steady state is established.

But problems with the steady-state methods exist: (i) a large number of pulses are required to establish a steady state. For example, with a  $5^\circ$  flip-angle pulse, a 5-ms recovery delay, and a sample of spin-lattice relaxation time  $T_1 = 500$  ms, more than 300 RF pulses are required for the magnetization to reach 95% of the steady-state value [2]. Not only is it a waste of time in fast imaging [3–5], but also the excessive RF power deposition to the imaged subject can be a serious concern. (ii) Further, a preparation sequence is usually added ahead of the pulse train to make the magnetization sensitive to relaxations, diffusion, etc., or to perform slice selection or phase encoding to the magnetization. For example, a preparation sequence for spatial encoding makes the magnetization a function of spatial coordinates so that the NMR signals from the pulse train will contain the coordinate information. But, as it will be shown in Section 4.1, the preparation sequence is useless if the magnetization reaches the steady state because in the steady state the magnetization is no longer a function of the parameters that the preparation sequence is designed for. (iii) There are situations where the pulse train must be segmented to insert, e.g., lipid NMR signal suppression [6], which adds to the difficulties of maintaining a steady state. (iv) Many dynamical systems simply do not have steady-state magnetization. For example, a selected slice of flow does not stay for steady state.

Methods utilizing transient-state magnetization do not have the above drawbacks but the fluctuating magnetization results in unevenly weighted signals and the spin-lattice relaxation degrades the effect of the preparation sequence [2,7–10]. To obtain accurate results from transient-state methods, the present work is interested in solving these problems. Because the application of an RF pulse couples the longitudinal and the transverse magnetization, a full theoretical analysis will be very involved except for pulse trains in which the phase of the magnetization is balanced [11]. Hence, the present work will focus on the condition that the transverse magnetization is zero at the time each RF pulse is applied. The condition is satisfied if the transverse-relaxation time is much shorter than the repetition delay or the flip angle of the RF pulses is low. First, a mathematical expression describing the NMR signals of a pulse train in the transient state is derived (Section 3). The mathematical expression for the NMR signals can be grouped into two terms; one depends on the initial

magnetization, which contains the information encoded by the preparation sequence, and the other does not depend on the initial magnetization but is the origin of the problems. Therefore by using signal averaging with different initial magnetization the problems can be eliminated as shown in Section 4. In addition, based on the theoretical analysis, a novel time-saving data acquisition method [12,13] for constructing a spin-lattice relaxation map ( $T_1$  map) is also introduced in this work (Section 4). The MRI pulse sequence used in this work for experimental examination and demonstration is described in Section 2 and detailed in Appendix A.

## 2. Method

The transient-state MRI method used experimentally in the present study is the rotating ultra-fast imaging sequence (RUFIS) [14,15], which is depicted in Fig. 1. The preparation sequence performs slice selection and transverse-relaxation weighting. RUFIS contains a train of low flip-angle RF pulses. The direction of the read gradient,  $\theta$ , is set at a different value for each RF pulse so the free induction decays (FIDs) are radial samples in the image conjugate domain (the  $k$ -space). The image is reconstructed from the radial samples by filtered back-projection. At the end of RUFIS, an RF pulse is applied to acquire an auxiliary signal with zero read gradient. The details of RUFIS and the image reconstruction are given in Appendix A.

Unless otherwise noted, RUFIS in this work contained 64 RF pulses and scanned the  $k$ -space for  $\theta \in [0, 2\pi)$ . The read gradient was set to its first value 3 ms prior to the commencement of the pulse train to dephase the remaining transverse magnetization. The central 1-cm slice of the imaged subject was selected using a five-lobe sinc shape (i.e., truncated at  $\pm 3\pi$ ) RF pulse (1.55 ms long). The flip angle  $\alpha$  was  $6^\circ_{+x}$  or  $8^\circ_{+x}$  long. Each FID was sampled at one complex point per  $\mu$ s for 128 points. For the back-projection reconstruction, the  $k$ -space data set was equivalent to the discrete Fourier transformation matrix size of  $256 \times 256$ . The

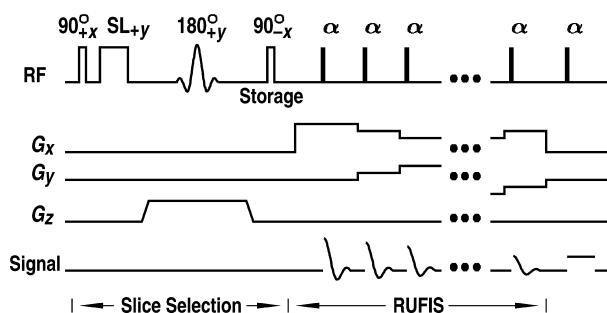


Fig. 1. Slice selective RUFIS. SL is a spin-lock pulse and  $\alpha$ 's are low flip-angle pulses.

NMR probe [15] and the gradient coils were in-house-made. The RF pulses, pre-emphasized gradient outputs, and the Zeeman-field ( $B_0$ ) compensation were generated by a digital console. All of the experiments were performed at 2.35 T.

### 3. Theory

In this section, the longitudinal magnetization with an arbitrary initial value and under the spin-lattice relaxation is calculated for a single pulse train and for a sequence of multiple pulse-trains; the latter is for an interesting application of relaxation-time mapping to be introduced in Section 4.3, which utilizes the pulse trains to produce a time series of images.

In this work,  $M_{\text{eq}}$  denotes the thermal equilibrium magnetization, and  $M_0$  the initial magnetization at the beginning of the first RF pulse.

#### 3.1. Single RF pulse train

Consider a train of uniformly spaced RF pulses. Let  $M_{z(n)}(\tau)$ ,  $n > 0$ , represent the longitudinal magnetization at time  $\tau$  following the  $n$ th RF pulse. The recovery of the longitudinal magnetization by the spin-lattice relaxation is described by

$$M_{z(n)}(\tau) = M_{z(n)}(0)e^{-\tau/T_1} + M_{\text{eq}}(1 - e^{-\tau/T_1}). \quad (1)$$

This equation is the solution to the Bloch equations in the absence of the RF field. If the transverse magnetization is zero (i.e., irreversibly relaxed) and if the pulse length is negligible, the effect of an RF pulse of flip angle  $\alpha$  is to produce a transverse component

$$M_{\perp(n)}(0) = M_{z(n-1)}(\Delta) \sin \alpha, \quad (2)$$

and to change the longitudinal component to be

$$M_{z(n)}(0) = M_{z(n-1)}(\Delta) \cos \alpha, \quad (3)$$

where  $\Delta$  is the time between the RF pulses.

The weight of the  $n$ th NMR signal in the pulse train is determined by  $M_{\perp(n)}(0)$ . Given the initial longitudinal magnetization  $M_0$ , one can obtain the expression for  $M_{\perp(n)}(0)$  by iteratively substituting Eqs. (1)–(3). For the first RF pulse,  $M_{\perp(1)}(0) = M_0 \sin \alpha$  by using Eq. (2) with  $M_{z(0)}(\Delta) = M_0$ . To calculate for the second RF pulse,  $M_{z(1)}(\Delta)$  is needed, which can be obtained by substituting Eq. (3) into Eq. (1); then from Eq. (2) with  $n = 2$ ,

$$M_{\perp(2)}(0) = [M_0 E_{1\Delta} \cos \alpha + M_{\text{eq}}(1 - E_{1\Delta})] \sin \alpha, \quad (4)$$

where  $E_{1\Delta} = e^{-\Delta/T_1}$ . Each iteration for the next RF pulse multiplies a factor  $(E_{1\Delta} \cos \alpha)$  and adds a term  $M_{\text{eq}}(1 - E_{1\Delta})$  to the expression in the square brackets of Eq. (4). Finally, the desired expression for  $M_{\perp(n)}(0)$  is obtained

$$M_{\perp(n)}(0) = \left\{ M_0 (E_{1\Delta} \cos \alpha)^{n-1} + M_{\text{eq}}(1 - E_{1\Delta}) \times [(E_{1\Delta} \cos \alpha)^{n-2} + (E_{1\Delta} \cos \alpha)^{n-3} + \cdots + 1] \right\} \sin \alpha, \quad (5)$$

or, with the formula of the geometric series sum,

$$M_{\perp(n)}(0) = \left[ M_0 (E_{1\Delta} \cos \alpha)^{n-1} + M_{\text{eq}}(1 - E_{1\Delta}) \frac{1 - (E_{1\Delta} \cos \alpha)^{n-1}}{1 - E_{1\Delta} \cos \alpha} \right] \sin \alpha. \quad (6)$$

A similar approach by iterative substitution can be found in [2]. Keep in mind that  $M_z$ ,  $M_{\perp}$ ,  $M_{\text{eq}}$ ,  $M_0$ , and  $T_1$  are functions of spatial coordinates if the subject is heterogeneous.

#### 3.2. Multiple RF pulse-trains

Consider a sequence of multiple trains; each train has  $N$  RF pulses as shown in Fig. 2. The first pulse train starts after a delay  $D_1$  from the time point at which  $M_0$  is specified, and the  $i$ th pulse train,  $i > 1$ , starts after a delay  $D_i$  from the end of the last train. The end of a train is defined as time  $\Delta$  past the last pulse of the train. In what follows,  $E_{1D_i} \equiv e^{-D_i/T_1}$ , and  $(i, n)$  in the subscript labels the quantities for the  $n$ th RF pulse of the  $i$ th pulse train.

For the first pulse train, Eq. (6) should be rewritten to include the spin-lattice relaxation during  $D_1$ . Therefore  $M_0$  in Eq. (6) is substituted by  $M_0 E_{1D_1} + M_{\text{eq}}(1 - E_{1D_1})$  as described by Eq. (1), that is

$$M_{\perp(1,n)}(0) = [M_0 (E_{1\Delta} \cos \alpha)^{n-1} E_{1D_1} + C_1(n)] \sin \alpha, \quad (7)$$

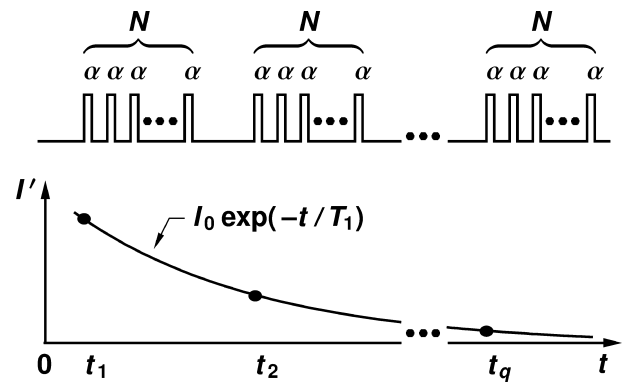


Fig. 2. Schematic representation of sampling a decay exponential curve of time constant  $T_1$  by a sequence of  $q$  RF pulse-trains (with eliminative averaging). An image is reconstructed from each train and represents a sample point on the curve.

where

$$C_1(n) = M_{\text{eq}}(1 - E_{1D_1})(E_{1A} \cos \alpha)^{n-1} + M_{\text{eq}}(1 - E_{1A}) \times \frac{1 - (E_{1A} \cos \alpha)^{n-1}}{1 - E_{1A} \cos \alpha}. \quad (8)$$

At the end of the first pulse train, the longitudinal magnetization is given by [cf. Eq. (1)]

$$M_{z(1,N)}(\Delta) = M_{z(1,N)}(0)E_{1A} + M_{\text{eq}}(1 - E_{1A}) \quad (9)$$

with  $M_{z(1,N)}(0)$  given by [cf. Eq. (7)]

$$M_{z(1,N)}(0) = \left[ M_0(E_{1A} \cos \alpha)^{N-1} E_{1D_1} + C_1(N) \right] \cos \alpha. \quad (10)$$

For the second pulse train,  $M_{\perp(2,n)}(0)$  can be calculated by reusing Eqs. (7) and (8) with  $M_{\perp(1,n)}(0)$  replaced by  $M_{\perp(2,n)}(0)$ ,  $E_{1D_1}$  replaced by  $E_{1D_2}$ , and  $M_0$  substituted by  $M_{z(1,N)}(\Delta)$  of Eq. (9). By iterating the calculation, a general formula for the  $n$ th RF pulse of the  $i$ th train,  $i > 1$ , is arrived:

$$M_{\perp(i,n)}(0) = \left[ M_0(E_{1A} \cos \alpha)^{(i-1)N+n-1} E_{1D_1} E_{1D_2} \cdots E_{1D_i} + C_i(n) \right] \sin \alpha \quad (11)$$

or

$$M_{\perp(i,n)}(0) = \left[ M_0(E_{1A} \cos \alpha)^{n-1} \cos^{(i-1)N} \alpha e^{-t_i/T_1} + C_i(n) \right] \sin \alpha, \quad (12)$$

with

$$C_i(n) = C_{i-1}(N)E_{1D_i}(E_{1A} \cos \alpha)^n + M_{\text{eq}}(1 - E_{1A}E_{1D_i}) \times (E_{1A} \cos \alpha)^{n-1} + M_{\text{eq}}(1 - E_{1A}) \frac{1 - (E_{1A} \cos \alpha)^{n-1}}{1 - E_{1A} \cos \alpha}, \quad (13)$$

where  $t_i$  is the time when the  $i$ th pulse train is started relative to the time point at which  $M_0$  is specified. If  $i = 1$ , Eqs. (7) and (8) are used. The expressions for  $C_i(n)$ 's are complicated, but one only needs to remember that they are not functions of  $M_0$ . For convenience, the first term in the square brackets of Eqs. (6), (7), and (12) is referred to as the  $M_0$  term and the second term as the *recovery term*.

## 4. Results and discussion

### 4.1. Steady-state magnetization revisited

Eq. (6) describes the transverse component of the magnetization as a function of the number of RF pulses  $n$ . The non-trivial steady state (SS) is achieved when  $n$  is infinitely large and then Eq. (6) becomes

$$M_{\perp}^{\text{SS}}(0) = M_{\text{eq}} \frac{1 - E_{1A}}{1 - E_{1A} \cos \alpha} \sin \alpha, \quad (14)$$

which agrees with the formula of Ernst et al. [1,16], who started from the recursion relation of the steady state (also see [17]). Note that the  $M_0$  dependence is gone. Consequently, if a preparation sequence is performed prior to the pulse train to encode information to the initial magnetization  $M_0$ , the signals acquired in the steady state do not carry the encoded information.

### 4.2. Transient-state imaging

Accurately recalling the information encoded to  $M_0$  from the transient-state signals is feasible only if the recovery term in Eq. (6) is negligible in comparison with the  $M_0$  term, which is not the case when  $n$  is large or  $T_1$  is short. Removal of the recovery term by post-processing is possible if  $M_{\text{eq}}$  and  $T_1$  are known a priori. But the determination of  $M_{\text{eq}}$  and  $T_1$ , both being functions of spatial coordinates, requires extra works and adds to the difficulties. In this section, an experimental technique that can separate the  $M_0$  and the recovery terms without post-processing is described, and the artifact due to the recovery term is demonstrated.

Consider two experiments: one is performed regularly, but the other is performed with the sign of  $M_0$  inverted. For pulse sequences similar to that shown in Fig. 1, the sign inversion can be accomplished by executing the storage RF pulse with the phase  $+x$ . Subtraction between the two data sets gives [cf. Eq. (6)]

$$\begin{aligned} \tilde{M}_{\perp(n)}(0) &\equiv M_{\perp(n)}(0)|_{M_0} - M_{\perp(n)}(0)|_{-M_0} \\ &= 2M_0(E_{1A} \cos \alpha)^{n-1} \sin \alpha. \end{aligned} \quad (15)$$

The recovery term is eliminated. Similarly, if the two data sets are added, only the recovery term remains. Then the two terms are separated.

To compare the recovery term and the  $M_0$  term in their magnitude as the number of pulses increases, a series of RUFIS experiments were performed; each experiment had a different number of RF pulses and a regular or an inverted  $M_0$ . The sample (detailed in Section 4.3) was composed of liquids of various  $T_1$  values in the range from 0.2 to 1.1 s. The direction of the RUFIS read gradient was held constant. By using the auxiliary signal at the end of RUFIS (see Fig. 1 and Section 2), the recovery-to- $M_0$ -term ratio was calculated. For  $n = 0, 16, 32, 48$ , and  $64$ , the ratios were 0.05, 0.10, 0.15, 0.19, and 0.24, respectively. The ratio will vary from study to study, e.g., the thinner the slice width and/or the shorter the  $T_1$ , the greater the ratio; nevertheless, the present example has shown that the recovery effect may not be negligible in transient-state methods.

According to Eq. (15), if an image is to be reconstructed from the NMR signals of the pulse train, each signal needs to be multiplied by the normalization factor  $(E_{1A} \cos \alpha)^{1-n}$  to even the signals. This factor is  $T_1$ -dependent; therefore, the normalization could be

impractical for heterogeneous subject because the NMR signal is in fact determined by the volume integral of the transverse magnetization. Nevertheless, if  $\Delta \ll T_1$  (e.g.,  $\Delta = 180 \mu\text{s}$  in this work and  $T_1 \sim 800 \text{ ms}$  is common in vivo), then  $E_{1A}$  can be replaced by one or by a representative constant of the imaged subject.

MRI with improved signal-to-noise ratio is usually achieved by averaging repeated experiments. As suggested by the above analysis, employing inverted  $M_0$  for half of the total number of repetitions can eliminate the spin-lattice relaxation effect and will be referred to as *eliminative averaging* in the following discussion in contrast to the regular *accumulative averaging* which has no  $M_0$  inversion. Fig. 3 shows two RUFIS images of a kiwi fruit with  $T_1$  determined in a separate experiment to be in the range from 0.9 to 1.4 s. The read gradient was  $G_{\text{read}} = 5.51 \text{ G/cm}$ . A delay was inserted in the preparation sequence such that the spin-lock and the storage RF pulses were separated by 102 ms to let  $M_0$  be modulated by the transverse relaxation. Hence the images

were meant to be transverse-relaxation ( $T_2^*$ ) weighted. For each image, the range of the intensity was linearly scaled to cover the full gray scale. As can be seen in the figure, without eliminating the spin-lattice relaxation effect, the accumulative-averaging image can hardly show proper contrast. The present MRI system has an RF leak [10]. The Fourier spectra of the signals from an empty probe have a sharp peak at 57 kHz off the NMR frequency. The leak signal is responsible for the swirly scars at the outline of the fruit in the accumulative-averaging image. It is interesting to note that, since eliminative averaging is equivalent to a phase cycling technique, the eliminative averaging can effectively remove the leak signal.

The magnitude of the spin-lattice relaxation effect was not expected for the pulse train method FLASH [18] in which a steady-state expression was used to analyze the transient-state method. However, Coremans et al. [8] found that the spin-lattice relaxation causes serious error in quantitative FLASH imaging but can be eliminated by a subtraction sequence, i.e., eliminative averaging.

Fig. 4 further shows a series of  $T_2^*$  weighted images using RUFIS with eliminative averaging. Each image was acquired with a different delay in the preparation sequence. These images suggest that RUFIS with eliminative averaging is capable of delivering the effect of the preparation sequence.

#### 4.3. Time-saving $T_1$ -map imaging

Eq. (12) describes the transient-state magnetization of a sequence of multiple pulse-trains. If two experiments are performed for eliminative averaging, the resulting NMR signal for the  $n$ th RF pulse in the  $i$ th pulse train is proportional to the volume integral of

$$\tilde{M}_{\perp(i,n)}(0) = 2M_0(E_{1A} \cos \alpha)^{n-1} \cos^{(i-1)N} \alpha e^{-t_i/T_1} \sin \alpha. \quad (16)$$

Recall that  $t_i$  is the time when the  $i$ th pulse train is started relative to the time at which  $M_0$  is specified, and  $N$  is the number of RF pulses in a train. If the signals in a pulse train, after normalization, are used to reconstruct one image  $I_i$ , then Eq. (16) implies

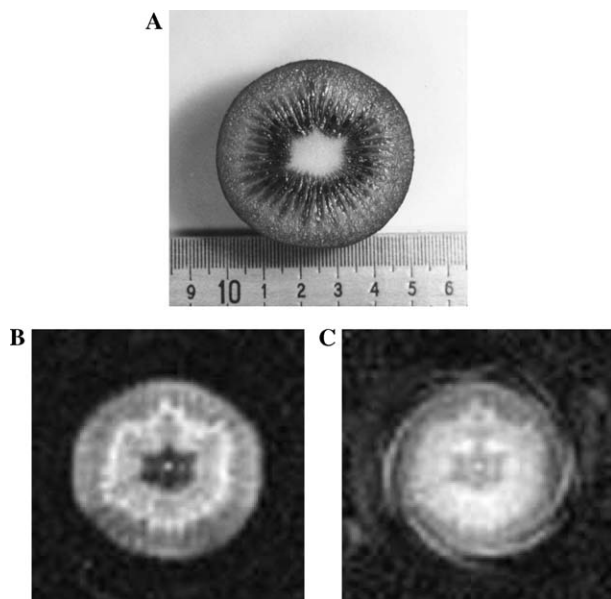


Fig. 3. Photograph (A) and RUFIS MRI images (B and C; matrix size:  $45 \times 45$ ) of a kiwi fruit. For (B), eliminative averaging was used to remove the spin-lattice relaxation effect while (C) was obtained by regular (accumulative) averaging.

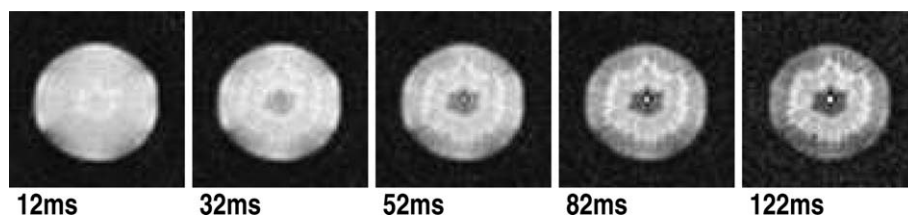


Fig. 4. Transverse-relaxation weighted images of a kiwi fruit using RUFIS with eliminative averaging. The labels indicate the time span of the preparation sequence. The image size is  $45 \times 45$ .

$$I_i = I_0 \cos^{(i-1)N} \alpha e^{-t_i/T_1} \quad (17)$$

or

$$I'_i = I_0 e^{-t_i/T_1}, \quad (18)$$

where  $I'_i = I_i \cos^{(1-i)N} \alpha$ . Eq. (18) is an exponential decay with a time constant of the sample's  $T_1$ ; it suggests a time-saving method for acquiring data to produce a  $T_1$  map as illustrated in Fig. 2, in which an imaging sequence of multiple pulse-trains is performed and repeated for eliminative averaging. The image reconstructed from the  $i$ th pulse train represents the sample point on the decay curve at time  $t_i$ . Then a  $T_1$  map can be obtained by fitting these images to decay exponential functions. If the pulse train is much shorter than  $T_1$ , the imaging sequence can accommodate as many pulse trains as the signal-to-noise ratio allows, and one can obtain multiple sample points of the decay curve in one batch. Conventional methods such as the inversion-recovery would obtain only one sample point at a time.

Fig. 5 shows  $T_1$  maps obtained from the suggested method. Following the slice selection, the imaging sequence consisted of six RUFIS trains with a constant inter-train delay of 180 ms. Each RUFIS train took 12 ms. The imaged subject (also used in Section 4.2) was constructed with seven glass vials; each of them is of 4.5-cm height and 1.2-cm diameter and was filled with liquid of  $T_1$  value shown in Table 1. Fig. 5 shows a  $T_1$  map from 16 repetitions with a delay of 8 s before each repetition to allow recovery of the magnetization. In the  $T_1$  map, the variation of the intensity among the vials does follow the reference values shown in Table 1. Each reference value was obtained by individually measuring a 0.2-cm<sup>3</sup> sample using the inversion-recovery, or 180°- $\tau$ -90°, pulse sequence with a solenoid coil and the same MRI system. For each liquid, 10 sample points were obtained for curve fitting; experimentally, each point took four repetitions with 8-s repetition delays.

To compare the  $T_1$  map with the reference values, the  $T_1$  values of the pixels belonging to the same vial are

Table 1

Spin-lattice relaxation times (s) measured by multiple pulse-train MRI and the reference values by the conventional spectroscopy technique

Sample	Reference value	MRI	
		A	B
1	1.02	1.07 (0.20)	1.01 (0.25)
2	0.62	0.66 (0.11)	0.64 (0.15)
3	0.46	0.53 (0.05)	0.56 (0.11)
4	0.37	0.37 (0.04)	0.37 (0.07)
5	0.31	0.33 (0.03)	0.35 (0.06)
6	0.27	0.27 (0.02)	0.28 (0.06)
7	0.21	0.22 (0.03)	0.22 (0.04)

The numbers in the parentheses are the standard deviations. Columns A and B are obtained from the 16-scan and the 2-scan experiment, respectively.

averaged, and the average is tabulated in Table 1 together with the standard deviation in the parentheses. As shown in column A, the MRI results are in very good agreement with the reference values. Even with only two scans, the present method can produce reasonably accurate results as shown in Fig. 5 and in column B of Table 1. The two-scan experiment took only 4.2 s, including a 2-s recovery delay. According to Eq. (18), the  $i$ th image needs to be multiplied by the factor  $\cos^{(1-i)N} \alpha$  before the exponential-curve fitting. Therefore, the error in the images obtained from later pulse-trains can be amplified by the multiplication; the accuracy of the flip angle  $\alpha$  can also affect the measurement if the flip angle is large. The two-scan experiment has lower signal-to-noise ratio and, therefore, has larger standard deviations as expected.

Other multiple-point  $T_1$ -map techniques [19–23] and the inversion-recovery method require one or more of the following conditions:  $M_0 = 0$ ,  $-M_{eq}$ , or  $-M_z^{SS}$ , or the determination of  $M_{eq}$  or  $M_z^{SS}$ . The present method does not have these requirements.

#### 4.4. Remarks

In this work, the two sets of experiments for eliminative averaging are performed with initial magnetization  $M_0$  and  $-M_0$ , respectively; the latter is achieved by an inversive RF pulse. However, if the second set starts with  $-\beta M_0$ , where  $\beta$  is an arbitrary constant, the coefficient  $2M_0$  in Eqs. (15) and (16) becomes  $(1 + \beta)M_0$  and the eliminative averaging still produces a decay exponential function with the time constant unchanged. Thus, if the measurement concerns only the  $T_1$  value, the second experiment does not require the magnetization exactly equal  $-M_0$ . Therefore, an inversive RF pulse not perfectly at the theoretical flip-angle is tolerable as long as the transverse magnetization is zero prior to the pulse train. Similarly, full recovery of the magnetization before the second experiment, although resulting in higher signal-to-noise ratio, is not necessary.

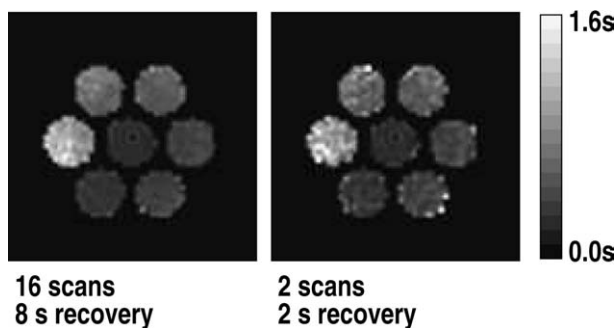


Fig. 5. Spin-lattice relaxation ( $T_1$ ) maps by multiple pulse-train MRI. The sample liquid at the center is numbered 7; the others, from the 9-o'clock position and clockwise, are numbered 1–6.

To sample the exponential curve of Eq. (18), the present work uses a sequence of multiple pulse-trains. As it can be seen in Fig. 2 with  $N = 1$ , the concept can be extended to MRI methods which acquire one image with a single RF pulse (e.g., echo planar imaging, spiral scan, etc.). One can verify this with Eqs. (16) and (17) by setting  $N = 1$  and  $n = 1$ , and finds that Eq. (18) is still preserved.

## 5. Conclusion

Theoretically, the longitudinal magnetization with arbitrary initial value  $M_0$  and under the spin-lattice relaxation is calculated for a signal RF pulse train and for a sequence of multiple RF pulse-trains. It is assumed that the transverse magnetization is zero when each RF pulse is applied. The general expressions of the magnetization can be grouped into two terms: a decay term, which is proportional to  $M_0$ , and a recovery term, which has no  $M_0$  dependence but is strongly affected by the spin-lattice relaxation.

Experimentally, the results from the rotating ultra-fast imaging sequence (RUFIS) suggest that in methods using transient-state magnetization the contribution of spin-lattice relaxation to the NMR signals is not negligible. The relaxation can seriously degrade the function of the preparation sequence for slice selection, contrast weighting, phase encoding, etc. Nevertheless, the spin-lattice relaxation effect can be removed by eliminative averaging in which half of the total number of repetitions are performed with an inverted  $M_0$  and the data are subtracted from that of the other half repetitions.

The present work introduces a time-saving data acquisition method for constructing the  $T_1$  map. The method has two features: (i) By using eliminative averaging, the curve to fit for the  $T_1$  value is a decay exponential function Eq. (18). Conventional transient-state techniques, such as inversion-recovery, will involve fitting a recovery exponential function, e.g., Eq. (1), which requires the determination of the equilibrium magnetization  $M_{eq}$  and/or  $M_0 = 0$ , or  $-M_{eq}$ . The present method is not subject to these requirements and the result is less susceptible to the accuracy of the flip angle of the inversion RF pulse. (ii) The decay exponential is sampled by using a sequence of multiple pulse-trains. An image is reconstructed from each pulse train and represents the sample point at the time when that pulse train is started (see Fig. 2). Hence a single imaging sequence can yield multiple sample points needed for data fitting to obtain the  $T_1$  map. Conventional techniques require repeating the imaging sequence for various values of the recovery delay but each repetition obtains only one sample point. The multiple pulse-train method is tested by using RUFIS and is found to be effective to produce accurate  $T_1$  maps.

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## Appendix A

This appendix describes in detail the NMR imaging pulse sequence shown in Fig. 1. The first  $90^\circ_{+x}$  pulse turns the magnetization into the transverse plane; if the flip angle is not at perfect  $90^\circ$ , the spin-lock pulse, implemented as a long (3 ms) rectangular RF pulse, “dephases” the  $zx$ -plane component of the magnetization. The slice gradient  $G_z$  and the shaped  $180^\circ_{+y}$  RF pulse dephase the transverse magnetization and refocus only that inside the desired slice. The succeeding  $90^\circ_{-x}$  storage pulse turns the transverse magnetization back along the longitudinal axis. Depending on the purpose of the study, the preparation sequence may include additional segments for spatial or velocity encoding; the encoded information will be stored as the longitudinal magnetization by the storage pulse. The main part in Fig. 1 is the rotating ultra-fast imaging sequence (RUFIS) [14,15]. RUFIS contains a train of low flip-angle RF pulses (flip angle  $\alpha$ ) with the read gradient given by  $G_x = G_{read} \cos \theta$  and  $G_y = G_{read} \sin \theta$ ; the FID signal is recorded immediately after each RF pulse. The polar direction of the read gradient,  $\theta$ , is kept constant during the RF transmission and signal reception period and is set to a new value before the application of the next RF pulse. The angle  $\theta$  varies through equal-spaced values in  $[0, 2\pi)$  so the image conjugate domain (the  $k$ -space) is sampled radially by the FIDs. The FIDs need to be normalized by the factor  $(E_{1A} \cos \alpha)^{1-n}$  as discussed after Eq. (15). In a RUFIS train, the gradient outputs  $G_x$  and  $G_y$  step gradually; therefore the induced eddy current is expected to be small. Other features of RUFIS include that acquisition of the FID eliminates the time needed to refocus an echo, and the information encoded in the preparation sequence is preserved in the longitudinal rather than the transverse magnetization until recalled by each RF pulse. Both reduce the time the magnetization stays in the transverse plane, making RUFIS a method less susceptible to de-coherence due to field inhomogeneity and time varying factors such as diffusion, motion, flow, etc.

At the end of RUFIS, the read gradient is set to zero; then an additional  $\alpha$  RF pulse is executed to produce an auxiliary NMR signal (Fig. 1) for the purpose of the

signal extrapolation of the initial portion of each RUFIS FID. The initial portion cannot be recorded because the receiver is blanked for the RF transmission. When being tilted by the RF pulse under a non-zero read gradient, the magnetization is also precessing relative to the rotating frame, generating imaging signal; but the signal cannot be recorded by the blanked receiver. The initial portions must be recovered before the FIDs can be used in the image reconstruction. Recently, a simple extrapolation formula has been given by Hsu and Lowe [15] to treat the problem of incomplete signal acquisition. It was found that if the data point at the origin of the FID is known and is entered to the extrapolation computation, the extrapolated signal is almost identical to the actual FID. The auxiliary signal is to represent the data at the FID origin since the signal is acquired at zero gradient and all decay mechanisms of the NMR signal such as transverse relaxation and field inhomogeneity are negligible during the very short period of the receiver blanking and signal acquisition (e.g.,  $\sim 150 \mu\text{s}$ ). In this work, the auxiliary signal was normalized the same way as the RUFIS FIDs; then the average value  $S_{\text{aux}}$  of the data points of the normalized auxiliary signal was calculated. However,  $S_{\text{aux}}$  cannot be used directly for the origin of the normalized RUFIS FIDs because, while the auxiliary RF pulse flips the magnetization uniformly across the imaged subject, the flip angle by the RF pulses of RUFIS is not uniform due to the non-zero read gradient. For a low flip-angle RF pulse, the spatial profile of the flip angle can be approximated by a sinc function,  $\text{sinc}(x) \equiv \sin(x)/x$ , along the read gradient direction. For a slice with a circular plane and centered at the gradient center, the value  $S_0$  for the origin of the FID with non-zero gradient is

$$S_0 = S_{\text{aux}} \frac{\int_0^R \text{sinc}(\frac{L}{2}r) \sqrt{R^2 - r^2} dr}{\int_0^R \sqrt{R^2 - r^2} dr}, \quad (\text{A.1})$$

where  $L$  is the RF pulse length divided by the receiver sampling period and  $R$  is the radius of the imaged subject divided by the width corresponding to the Nyquist frequency and multiplied by  $\pi$ . The integral in the denominator is simply the area of a circle and that in the numerator is the area weighted by a sinc function along the diameter. More symmetric and homogeneous is the imaged subject, more accurately  $S_0$  approximates the origin of the RUFIS FIDs.

In Hsu and Lowe [15], a pulse sequence was designed to acquire a stimulated spin echo (SSE) to examine the FID. In this work, the SSE pulse sequence determined the number of blanked data points in the FID, which is needed in the extrapolation computation. This number is considered a constant when the identical pulse sequence and parameters are used on the same MRI system. Therefore, once the number of blanked data is determined, the SSE pulse sequence is not repeated for

routine RUFIS experiments. The signal recovery and the image reconstruction from RUFIS FIDs are detailed in Hsu and Lowe [15].

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